Question
Given a function $f : A \to \mathbf{R}$, define a new function $-f : A \to \mathbf{R}$ by $(-f)(a) = -f(a)$. Prove that $\inf(-f) = -\sup(f)$.

Answer
Let $c = \sup(f)$, so that $c = \sup\{f(a) \mid a \in A\}$. In particular, $c \geq f(a)$ for all $a \in A$, and if u is any number satisfying $u \geq f(a)$ for all $a \in A$, then $u \leq c$. Multiplying by -1, we see that $-c \leq -f(a)$ for all $a \in A$ and that if s is any number satisfying $s \leq -f(a)$ for all $a \in A$, then $s \geq -c$. However, this is exactly the definition that $-c = \inf(-f)$, as desired.